

Cycles on Shimura varieties via geometric Satake

Fall 2024 Learning Seminar

Time: Thursdays 1:30 PM - 3:30 PM *Location:* Room 2-255 at MIT

We discuss some aspects of the paper [XZ17] by Liang Xiao and Xinwen Zhu.¹ Here are some further references:

- Lecture series by Liang Xiao, with notes and recordings: <https://ymsc.tsinghua.edu.cn/en/info/1050/2426.htm>.
- “Cycles on Shimura varieties via Geometric Satake, Example” by Xiao and Zhu: <http://faculty.bicmr.pku.edu.cn/~lxiao/example-new.pdf>.
- ARGOS seminar outline on Xiao–Zhu: https://www.math.uni-bonn.de/ag/algeom/veranstaltungen/ARGOS/ARGOS_WS1718.pdf.

-
- (1) (*Mar. 14*) Statement of the main result Section 1 of [XZ17], (see also Xiao’s Tsinghua lecture 1).
 - (2) (*Mar. 21*) Examples for the main theorem in Xiao–Zhu. See the first part of Xiao’s Tsinghua lecture 2, and examples [TX19] and [here](#).
 - (3) (*Apr. 4*) Basics on affine Grassmannian and statements of geometric Satake equivalence (see [Zhu16b], [Zhu18], Xiao’s Lecture 2).
 - (4) (*Apr. 11*) Examples and proofs of geometric Satake, relationship with classical Satake. (see [Zhu16b], [Zhu18], [XZ17] 3.4-3.5, Xiao’s Lecture 3)
 - (5) (*Apr. 18*) Basics on affine Deligne-Lusztig varieties, definitions and dimension formulae ([He18]). Their relation to Rapoport-Zink spaces ([Zhu16a]).
 - (6) (*Apr. 23*)²Fine structure on ADLV. Irreducible components of affine Deligne-Lusztig varieties ([XZ17] Section 4, Xiao’s Lecture 4).
 - (7) (*May. 2*) Moduli of local Shtukas and cohomological correspondence between them.
 - (8) (*May. 9*)

¹We will skip March 14 (IAS workshop) and March 28 (Spring break).

²Note that April 23 is a *Tuesday*. We will also meet at a different time and location for this day: Room 2-449 from 1 PM - 3 PM. This is due to a room scheduling conflict for our usual time. Our usual pre-seminar lunch will move to 12:15 PM for this week.

References

- [He18] Xuhua He. *Some results on affine Deligne-Lusztig varieties*. 2018. arXiv: [1807.02158 \[math.AG\]](#).
- [TX19] Yichao Tian and Liang Xiao. “Tate cycles on some quaternionic Shimura varieties mod p ”. In: *Duke Mathematical Journal* 168.9 (June 2019).
- [XZ17] Liang Xiao and Xinwen Zhu. *Cycles on Shimura varieties via geometric Satake*. 2017. arXiv: [1707.05700v1 \[math.AG\]](#).
- [Zhu16a] Xinwen Zhu. *Affine Grassmannians and the geometric Satake in mixed characteristic*. 2016. arXiv: [1407.8519 \[math.AG\]](#).
- [Zhu16b] Xinwen Zhu. *An introduction to affine Grassmannians and the geometric Satake equivalence*. 2016. arXiv: [1603.05593 \[math.AG\]](#).
- [Zhu18] Xinwen Zhu. *Geometric Satake, categorical traces, and arithmetic of Shimura varieties*. 2018. arXiv: [1810.07375 \[math.AG\]](#).